Problem 20

In each of Problems 19 and 20, verify the given equation.

$$\sum_{k=0}^{\infty} a_{k+1}x^k + \sum_{k=0}^{\infty} a_k x^{k+1} = a_1 + \sum_{k=1}^{\infty}(a_{k+1} + a_{k-1})x^k$$

Solution

Begin with the left side.

$$\sum_{k=0}^{\infty} a_{k+1}x^k + \sum_{k=0}^{\infty} a_k x^{k+1}$$

Make it so that both sums have x^k: substitute $k = n$ in the first one and substitute $n = k + 1$ in the second one.

$$\sum_{n=0}^{\infty} a_{n+1}x^n + \sum_{n-1=0}^{\infty} a_{n-1}x^n$$

Solve for n.

$$\sum_{n=0}^{\infty} a_{n+1}x^n + \sum_{n=1}^{\infty} a_{n-1}x^n$$

Write out the first term of the first sum so that it starts at $n = 1$ like the second sum.

$$a_1x^0 + \sum_{n=1}^{\infty} a_{n+1}x^n + \sum_{n=1}^{\infty} a_{n-1}x^n$$

Combine the two sums now.

$$a_1 + \sum_{n=1}^{\infty}(a_{n+1}x^n + a_{n-1}x^n)$$

Factor x^n.

$$a_1 + \sum_{n=1}^{\infty}(a_{n+1} + a_{n-1})x^n$$

As n is only a dummy index, it can be replaced with k.

$$a_1 + \sum_{k=1}^{\infty}(a_{k+1} + a_{k-1})x^k$$