Problem 25

In each of Problems 21 through 27, rewrite the given expression as a sum whose generic term involves x^n.

$$\sum_{m=2}^{\infty} m(m-1)a_m x^{m-2} + x\sum_{k=1}^{\infty} k a_k x^{k-1}$$

Solution

Bring x into the summand.

$$\sum_{m=2}^{\infty} m(m-1)a_m x^{m-2} + \sum_{k=1}^{\infty} k a_k x^{k}$$

Substitute $n = m - 2$ in the first sum and substitute $k = n$ in the second sum.

$$\sum_{n+2=2}^{\infty} (n+2)(n+1)a_{n+2} x^{n} + \sum_{n=1}^{\infty} n a_n x^{n}$$

Solve for n.

$$\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} x^{n} + \sum_{n=1}^{\infty} n a_n x^{n}$$

Because of n, the second sum can be started at $n = 0$.

$$\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} x^{n} + \sum_{n=0}^{\infty} n a_n x^{n}$$

Combine the two sums.

$$\sum_{n=0}^{\infty} [(n+2)(n+1)a_{n+2} x^{n} + n a_n x^{n}]$$

Factor x^n.

$$\sum_{n=0}^{\infty} [(n+2)(n+1)a_{n+2} + n a_n] x^{n}$$