Problem 3

In each of Problems 1 through 8, determine the radius of convergence of the given power series.

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$$

Solution

Apply the ratio test to determine the condition in which the provided series converges.

$$\begin{aligned}
\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty}\left|\frac{\frac{x^{2(n+1)}}{(n+1)!}}{\frac{x^{2n}}{n!}}\right| \\
&= \lim_{n\to\infty}\left|\frac{n!}{(n+1)!}\frac{x^{2n+2}}{x^{2n}}\right| \\
&= \lim_{n\to\infty}\left|\frac{1}{n+1}x^2\right| \\
&= \lim_{n\to\infty}\frac{1}{n+1}x^2 \\
&= 0x^2 \\
&= 0
\end{aligned}$$

According to this test, the series is

$$\begin{cases} \text{convergent} & \text{if } 0x^2 < 1 \\ \text{unknown} & \text{if } 0x^2 = 1 \\ \text{divergent} & \text{if } 0x^2 > 1 \end{cases}.$$

From the condition of convergence, which can also be written as $|x| < \sqrt{1/0} = \infty$, or $-\infty < x < \infty$, we see that the center of convergence is at $x = 0$ and the radius of convergence is ∞.