

Problem 26

In each of Problems 23 through 28, plot several partial sums in a series solution of the given initial value problem about $x = 0$, thereby obtaining graphs analogous to those in Figures 5.2.1 through 5.2.4.

$$(4 - x^2)y'' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 1; \quad \text{see Problem 10}$$

Solution

In Problem 10 the general solution was found to be

$$y(x) = a_0 \left(1 - \frac{x^2}{4} \right) + a_1 \left(x - \frac{x^3}{12} - \frac{x^5}{240} - \frac{x^7}{2240} - \dots \right).$$

Differentiate it with respect to x .

$$y'(x) = a_0 \left(-\frac{2x}{4} \right) + a_1 \left(1 - \frac{3x^2}{12} - \frac{5x^4}{240} - \frac{7x^6}{2240} - \dots \right).$$

Now apply the initial conditions, $y(0) = 0$ and $y'(0) = 1$, to determine a_0 and a_1 .

$$y(0) = a_0 = 0$$

$$y'(0) = a_1 = 1$$

Therefore,

$$y(x) = x - \frac{x^3}{12} - \frac{x^5}{240} - \frac{x^7}{2240} - \dots$$

Below is a plot of the various partial sums versus x .

