Problem 24

In each of Problems 23 through 28, plot several partial sums in a series solution of the given initial value problem about $x = 0$, thereby obtaining graphs analogous to those in Figures 5.2.1 through 5.2.4.

$$(2 + x^2)y'' - xy' + 4y = 0, \qquad y(0) = 1, \quad y'(0) = 0; \qquad \text{see Problem 6}$$

Solution

In Problem 6 the general solution was found to be

$$y(x) = a_0\left(1 - x^2 + \frac{x^4}{6} - \frac{x^6}{30} + \cdots\right) + a_1\left(x - \frac{x^3}{4} + \frac{7x^5}{160} - \frac{19x^7}{1920} + \cdots\right).$$

Differentiate it with respect to x.

$$y'(x) = a_0\left(-2x + \frac{2x^3}{3} - \frac{x^5}{5} + \cdots\right) + a_1\left(1 - \frac{3x^2}{4} + \frac{7x^4}{32} - \frac{133x^6}{1920} + \cdots\right)$$

Now apply the initial conditions, $y(0) = 1$ and $y'(0) = 0$, to determine a_0 and a_1.

$$y(0) = a_0 = 1$$
$$y'(0) = a_1 = 0$$

Therefore,

$$y(x) = 1 - x^2 + \frac{x^4}{6} - \frac{x^6}{30} + \cdots.$$

Below is a plot of the various partial sums versus x.