

Problem 1

In each of Problems 1 through 4, determine $\phi''(x_0)$, $\phi'''(x_0)$, and $\phi^{(4)}(x_0)$ for the given point x_0 if $y = \phi(x)$ is a solution of the given initial value problem.

$$y'' + xy' + y = 0; \quad y(0) = 1, \quad y'(0) = 0$$

Solution

Solve the ODE for y'' .

$$y'' = -xy' - y \tag{1}$$

Plug in $x = 0$.

$$y''(0) = -(0)y'(0) - y(0) = -1$$

Differentiate both sides of equation (1) with respect to x .

$$y''' = -y' - xy'' - y' \tag{2}$$

Plug in $x = 0$.

$$y'''(0) = -y'(0) - (0)y''(0) - y'(0) = 0$$

Differentiate both sides of equation (2) with respect to x .

$$y^{(4)} = -y'' - y'' - xy''' - y''$$

Plug in $x = 0$.

$$y^{(4)}(0) = -y''(0) - y''(0) - (0)y'''(0) - y''(0) = 1 + 1 + 1 = 3$$