Problem 4

In each of Problems 1 through 4, determine $\phi''(x_0)$, $\phi'''(x_0)$, and $\phi^{(4)}(x_0)$ for the given point x_0 if $y = \phi(x)$ is a solution of the given initial value problem.

$$y'' + x^2 y' + (\sin x)y = 0; \qquad y(0) = a_0, \quad y'(0) = a_1$$

Solution

Solve for y''.

$$y'' = -x^2 y' - (\sin x)y \tag{1}$$

Plug in $x = 0$.

$$y''(0) = 0$$

Differentiate both sides of equation (1) with respect to x.

$$y''' = -2xy' - x^2 y'' - (\cos x)y - (\sin x)y' \tag{2}$$

Plug in $x = 0$.

$$y'''(0) = -y(0) = -a_0$$

Differentiate both sides of equation (2) with respect to x.

$$y^{(4)} = -2y' - 2xy'' - 2xy'' - x^2 y''' + (\sin x)y - (\cos x)y' - (\cos x)y' - (\sin x)y''$$

Plug in $x = 0$.

$$\begin{aligned} y^{(4)}(0) &= -2y'(0) - 2(0)y''(0) - 2(0)y''(0) - (0)y'''(0) + (0)y(0) - (1)y'(0) - (1)y'(0) - (0)y''(0) \\ &= -2a_1 - a_1 - a_1 \\ &= -4a_1 \end{aligned}$$