

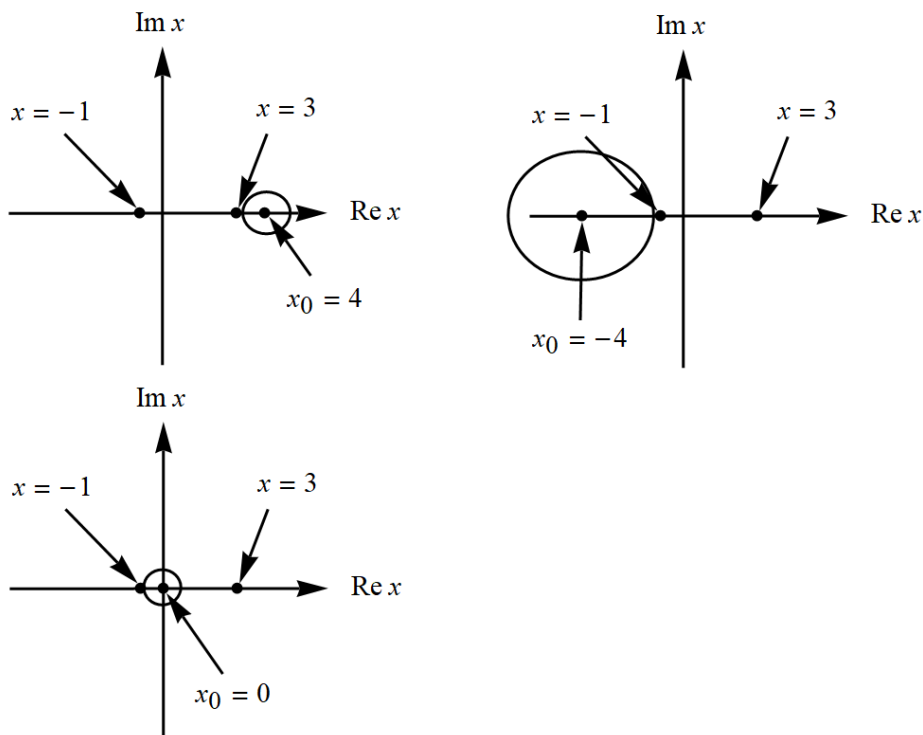
### Problem 6

In each of Problems 5 through 8, determine a lower bound for the radius of convergence of series solutions about each given point  $x_0$  for the given differential equation.

$$(x^2 - 2x - 3)y'' + xy' + 4y = 0; \quad x_0 = 4, \quad x_0 = -4, \quad x_0 = 0$$

#### Solution

The coefficient of  $y''$  is  $x^2 - 2x - 3$ . Its zeros are located at  $x = -1$  and  $x = 3$ . Plot these in the complex plane and expand a circle centered at  $x_0$  as much as possible before it intersects one of them.



If  $x_0 = 4$ , the lower bound for the radius of convergence is 1. If  $x_0 = -4$ , the lower bound for the radius of convergence is 3. If  $x_0 = 0$ , the lower bound for the radius of convergence is 1.