

Problem 23

In each of Problems 17 through 34, find all singular points of the given equation and determine whether each one is regular or irregular.

$$(x + 3)y'' - 2xy' + (1 - x^2)y = 0$$

Solution

The coefficient of y'' has a zero at $x = -3$, which means $x = -3$ is a singular point. To determine whether it is regular or irregular, divide both sides of the ODE by $x + 3$

$$y'' - \frac{2x}{x+3}y' + \frac{1-x^2}{x+3}y = 0$$

and compute the following limits.

$$\begin{aligned}\lim_{x \rightarrow -3} (x + 3) \left(-\frac{2x}{x + 3} \right) &= \lim_{x \rightarrow -3} (-2x) = 6 \\ \lim_{x \rightarrow -3} (x + 3)^2 \left(\frac{1 - x^2}{x + 3} \right) &= \lim_{x \rightarrow -3} (x + 3)(1 - x^2) = 0\end{aligned}$$

Because both limits as $x \rightarrow -3$ are finite, $x = -3$ is a regular singular point.