Problem 28

In each of Problems 17 through 34, find all singular points of the given equation and determine whether each one is regular or irregular.

$$xy'' + e^x y' + (3\cos x)y = 0$$

Solution

The coefficient of y'' has a zero at $x = 0$, which means $x = 0$ is a singular point. To determine whether it is regular or irregular, divide both sides of the ODE by x

$$y'' + \frac{e^x}{x}y' + \frac{3\cos x}{x}y = 0$$

and compute the following limits.

$$\lim_{x\to 0} x\left(\frac{e^x}{x}\right) = \lim_{x\to 0} e^x = 1$$
$$\lim_{x\to 0} x^2\left(\frac{3\cos x}{x}\right) = \lim_{x\to 0} 3x\cos x = 0$$

Because both limits as $x \to 0$ are finite, $x = 0$ is a regular singular point.