Problem 2

In each of Problems 1 through 10, find the inverse Laplace transform of the given function.

$$F(s) = \frac{4}{(s-1)^3}$$

Solution

Recall that one of the known Laplace transforms is

$$\mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}.$$

Write $F(s)$ in terms of this.

$$F(s) = 2\frac{2}{(s-1)^3}$$

Take the inverse Laplace transform of both sides.

$$\begin{aligned}
\mathcal{L}^{-1}\{F(s)\} &= \mathcal{L}^{-1}\left\{2\frac{2}{(s-1)^3}\right\} \\
f(t) &= 2\mathcal{L}^{-1}\left\{\frac{2}{(s-1)^3}\right\} \\
&= 2t^2e^t
\end{aligned}$$