# Problem 8

In each of Problems 1 through 10, find the inverse Laplace transform of the given function.

$$F(s) = \frac{8s^2 - 4s + 12}{s(s^2+4)}$$

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### Solution

Decompose the fraction partially.

$$\frac{8s^2 - 4s + 12}{s(s^2+4)} = \frac{A}{s} + \frac{Bs + C}{s^2+4}$$

Multiply both sides by $s(s^2+4)$.

$$8s^2 - 4s + 12 = A(s^2+4) + (Bs + C)s$$

Choose three random values for $s$ to get a system of equations for $A$, $B$, and $C$.

$$\begin{aligned} s = 0: &\qquad 12 = 4A \\ s = 1: &\qquad 16 = 5A + B + C \\ s = 2: &\qquad 36 = 8A + 4B + 2C \end{aligned}$$

Solving this system of equations yields $A = 3$, $B = 5$, and $C = -4$. As a result,

$$\begin{aligned} F(s) &= \frac{3}{s} + \frac{5s - 4}{s^2+4} \\ &= 3\frac{1}{s} + \frac{5s}{s^2+4} - \frac{4}{s^2+4} \\ &= 3\frac{1}{s} + 5\frac{s}{s^2+4} - 2\frac{2}{s^2+4}. \end{aligned}$$

Take the inverse Laplace transform to get $f(t)$, using the fact that the transform is a linear operator.

$$\begin{aligned} \mathcal{L}^{-1}\{F(s)\} &= \mathcal{L}^{-1}\left\{3\frac{1}{s} + 5\frac{s}{s^2+4} - 2\frac{2}{s^2+4}\right\} \\ f(t) &= 3\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + 5\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} - 2\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\} \\ &= 3e^0 + 5\cos 2t - 2\sin 2t \\ &= 3 + 5\cos 2t - 2\sin 2t \end{aligned}$$