Problem 10

In each of Problems 1 through 10, find the inverse Laplace transform of the given function.

$$F(s) = \frac{2s-3}{s^2+2s+10}$$

Solution

Complete the square in the denominator.

$$\begin{aligned} F(s) &= \frac{2s-3}{s^2+2s+1+10-1} \\ &= \frac{2s-3}{(s+1)^2+9} \end{aligned}$$

Make it so that the numerator has $s+1$ and write $F(s)$ in terms of known transforms.

$$\begin{aligned} F(s) &= \frac{2s+2-3-2}{(s+1)^2+9} \\ &= \frac{2(s+1)-5}{(s+1)^2+9} \\ &= \frac{2(s+1)}{(s+1)^2+9} - \frac{5}{(s+1)^2+9} \\ &= 2\frac{s+1}{(s+1)^2+9} - \frac{5}{3}\frac{3}{(s+1)^2+9} \end{aligned}$$

Take the inverse Laplace transform to get $f(t)$.

$$\begin{aligned} \mathcal{L}^{-1}\{F(s)\} &= \mathcal{L}^{-1}\left\{2\frac{s+1}{(s+1)^2+9} - \frac{5}{3}\frac{3}{(s+1)^2+9}\right\} \\ f(t) &= 2\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+9}\right\} - \frac{5}{3}\mathcal{L}^{-1}\left\{\frac{3}{(s+1)^2+9}\right\} \\ &= 2e^{-t}\cos 3t - \frac{5}{3}e^{-t}\sin 3t \end{aligned}$$