

Problem 28

The Laplace transforms of certain functions can be found conveniently from their Taylor series expansions.

- (a) Using the Taylor series for $\sin t$

$$\sin t = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!},$$

and assuming that the Laplace transform of this series can be computed term by term, verify that

$$\mathcal{L}\{\sin t\} = \frac{1}{s^2 + 1}, \quad s > 1.$$

- (b) Let

$$f(t) = \begin{cases} (\sin t)/t, & t \neq 0, \\ 1, & t = 0. \end{cases}$$

Find the Taylor series for f about $t = 0$. Assuming that the Laplace transform of this function can be computed term by term, verify that

$$\mathcal{L}\{f(t)\} = \arctan(1/s), \quad s > 1.$$

- (c) The Bessel function of the first kind of order zero, J_0 , has the Taylor series (see Section 5.7)

$$J_0(t) = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{2^{2n} (n!)^2}.$$

Assuming that the following Laplace transforms can be computed term by term, verify that

$$\mathcal{L}\{J_0(t)\} = (s^2 + 1)^{-1/2}, \quad s > 1$$

and

$$\mathcal{L}\{J_0(\sqrt{t})\} = s^{-1} e^{-1/(4s)}, \quad s > 0.$$