

Problem 29

Let

$$F(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

It is possible to show that as long as f satisfies the conditions of Theorem 6.1.2, it is legitimate to differentiate under the integral sign with respect to the parameter s when $s > a$.

- (a) Show that $F'(s) = \mathcal{L}\{-tf(t)\}$.
- (b) Show that $F^{(n)}(s) = \mathcal{L}\{(-t)^n f(t)\}$; hence differentiating the Laplace transform corresponds to multiplying the original function by $-t$.