

## Problem 32

In each of Problems 30 through 35, use the result of Problem 29 to find the Laplace transform of the given function;  $a$  and  $b$  are real numbers and  $n$  is a positive integer.

$$f(t) = t^n$$

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### Solution

The Laplace transform of a function  $f(t)$  is defined here as

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt.$$

Substitute the given function and evaluate the integral.

$$\begin{aligned} F(s) &= \int_0^{\infty} e^{-st} t^n dt \\ &= \int_0^{\infty} \left[ (-1)^n \frac{\partial^n}{\partial s^n} e^{-st} \right] dt \\ &= (-1)^n \frac{d^n}{ds^n} \int_0^{\infty} e^{-st} dt \\ &= (-1)^n \frac{d^n}{ds^n} \left( \frac{1}{-s} e^{-st} \Big|_0^{\infty} \right) \\ &= (-1)^n \frac{d^n}{ds^n} \left( \frac{1}{s} \right) \\ &= (-1)^n \frac{d^n}{ds^n} (s^{-1}) \\ &= (-1)^n [(-1)^n n! s^{-1-n}] \\ &= (-1)^{2n} \frac{n!}{s^{n+1}} \\ &= \frac{n!}{s^{n+1}} \end{aligned}$$