# Problem 9

In each of Problems 1 through 10, find the inverse Laplace transform of the given function.

$$F(s) = \frac{1-2s}{s^2+4s+5}$$

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### Solution

Complete the square in the denominator.

$$\begin{aligned} F(s) &= \frac{1-2s}{s^2+4s+4+5-4} \\ &= \frac{1-2s}{(s+2)^2+1} \end{aligned}$$

Make it so that the numerator has $s+2$ and write $F(s)$ in terms of known transforms.

$$\begin{aligned} F(s) &= \frac{4+1-2s-4}{(s+2)^2+1} \\ &= \frac{5-2(s+2)}{(s+2)^2+1} \\ &= \frac{5}{(s+2)^2+1} - 2\frac{s+2}{(s+2)^2+1} \end{aligned}$$

Take the inverse Laplace transform to get $f(t)$.

$$\begin{aligned} \mathcal{L}^{-1}\{F(s)\} &= \mathcal{L}^{-1}\left\{\frac{5}{(s+2)^2+1} - 2\frac{s+2}{(s+2)^2+1}\right\} \\ f(t) &= 5\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+1}\right\} - 2\mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+1}\right\} \\ &= 5e^{-2t}\sin t - 2e^{-2t}\cos t \end{aligned}$$