Problem 16

In each of Problems 13 through 18, find the Laplace transform of the given function.

$$f(t) = u_1(t) + 2u_3(t) - 6u_4(t)$$

Solution

The Laplace transform of a function $f(t)$ is defined here as

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt.$$

For the provided function in particular,

$$\begin{aligned} f(t) &= u_1(t) + 2u_3(t) - 6u_4(t) \\ &= H(t-1) + 2H(t-3) - 6H(t-4), \end{aligned}$$

we have

$$\begin{aligned} F(s) &= \int_0^\infty e^{-st}[H(t-1) + 2H(t-3) - 6H(t-4)]\,dt \\ &= \int_0^\infty e^{-st} H(t-1)\,dt + 2\int_0^\infty e^{-st} H(t-3)\,dt - 6\int_0^\infty e^{-st} H(t-4)\,dt \\ &= \int_1^\infty e^{-st}\,dt + 2\int_3^\infty e^{-st}\,dt - 6\int_4^\infty e^{-st}\,dt \\ &= \left(-\frac{1}{s}e^{-st}\right)\Big|_1^\infty + 2\left(-\frac{1}{s}e^{-st}\right)\Big|_3^\infty - 6\left(-\frac{1}{s}e^{-st}\right)\Big|_4^\infty \\ &= \left(\frac{1}{s}e^{-s}\right) + 2\left(\frac{1}{s}e^{-3s}\right) - 6\left(\frac{1}{s}e^{-4s}\right) \\ &= \frac{e^{-s} + 2e^{-3s} - 6e^{-4s}}{s} \end{aligned}$$