Problem 24

In each of Problems 19 through 24, find the inverse Laplace transform of the given function.

$$F(s) = \frac{e^{-s} + e^{-2s} - e^{-3s} - e^{-4s}}{s}$$

Solution

Apply the two transforms,

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \quad \text{and} \quad \mathcal{L}\{f(t-c)H(t-c)\} = F(s)e^{-cs},$$

together to solve this problem.

$$\begin{aligned}
f(t) &= \mathcal{L}^{-1}\{F(s)\} \\
&= \mathcal{L}^{-1}\left\{\frac{e^{-s} + e^{-2s} - e^{-3s} - e^{-4s}}{s}\right\} \\
&= \mathcal{L}^{-1}\left\{\frac{1}{s}e^{-s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s}e^{-2s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s}e^{-3s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s}e^{-4s}\right\} \\
&= (t-1)^0 H(t-1) + (t-2)^0 H(t-2) - (t-3)^0 H(t-3) - (t-4)^0 H(t-4) \\
&= H(t-1) + H(t-2) - H(t-3) - H(t-4) \\
&= u_1(t) + u_2(t) - u_3(t) - u_4(t)
\end{aligned}$$