Problem 28

In each of Problems 26 through 29, use the results of Problem 25 to find the inverse Laplace transform of the given function.

$$F(s) = \frac{1}{9s^2 - 12s + 3}$$

Solution

Observe that the denominator can be written in terms of $3s$.

$$F(s) = \frac{1}{(3s)^2 - 4(3s) + 3}$$

Factor the denominator.

$$F(s) = \frac{1}{[(3s) - 1][(3s) - 3]}$$

Partially decompose the fraction.

$$F(s) = \frac{-\frac{1}{2}}{(3s) - 1} + \frac{\frac{1}{2}}{(3s) - 3}$$

Apply the two transforms,

$$\mathcal{L}\{e^{at}\} = \frac{1}{s - a} \quad \text{and} \quad F(ks) = \mathcal{L}\left\{\frac{1}{k} f\left(\frac{t}{k}\right)\right\},$$

together to get $f(t)$.

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}\{F(s)\} \\ &= -\frac{1}{2}\left(\frac{1}{3}e^{t/3}\right) + \frac{1}{2}\left(\frac{1}{3}e^{3t/3}\right) \\ &= -\frac{1}{6}e^{t/3} + \frac{1}{6}e^{t} \\ &= \frac{1}{6}(e^{t} - e^{t/3}) \end{aligned}$$