Problem 29

In each of Problems 26 through 29, use the results of Problem 25 to find the inverse Laplace transform of the given function.

$$F(s) = \frac{e^2 e^{-4s}}{2s-1}$$

Solution

Rewrite the right side so that it's in terms of $2s-1$.

$$\begin{aligned} F(s) &= \frac{e^{2-4s}}{2s-1} \\ &= \frac{e^{-2(2s-1)}}{(2s-1)} \end{aligned}$$

Apply the three transforms,

$$\mathcal{L}\{1\} = \frac{1}{s} \quad \text{and} \quad \mathcal{L}\{f(t-c)H(t-c)\} = e^{-cs}F(s) \quad \text{and} \quad \mathcal{L}\left\{\frac{1}{a}e^{-bt/a}f\left(\frac{t}{a}\right)\right\} = F(as+b),$$

together to determine $f(t)$. Note that

$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s}\right\} = H(t-2),$$

so

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}\{F(s)\} \\ &= \mathcal{L}^{-1}\left\{\frac{e^{-2(2s-1)}}{(2s-1)}\right\} \\ &= \frac{1}{2}e^{t/2}H\left(\frac{t}{2}-2\right) \\ &= \frac{1}{2}e^{t/2}u_2\left(\frac{t}{2}\right). \end{aligned}$$