

## Problem 21

In each of Problems 19 through 24, find the inverse Laplace transform of the given function.

$$F(s) = \frac{2(s-1)e^{-2s}}{s^2 - 2s + 2}$$

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### Solution

Complete the square in the denominator.

$$\begin{aligned} F(s) &= 2 \frac{s-1}{s^2 - 2s + 1 + 2 - 1} e^{-2s} \\ &= 2 \frac{s-1}{(s-1)^2 + 1} e^{-2s} \end{aligned}$$

Apply the inverse Laplace transform now to get  $f(t)$ .

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}\{F(s)\} \\ &= \mathcal{L}^{-1}\left\{2 \frac{s-1}{(s-1)^2 + 1} e^{-2s}\right\} \\ &= 2\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2 + 1} e^{-2s}\right\} \\ &= 2e^{(t-2)} \cos(t-2)H(t-2) \\ &= 2e^{t-2} \cos(t-2)u_2(t) \end{aligned}$$