

Problem 23

In each of Problems 19 through 24, find the inverse Laplace transform of the given function.

$$F(s) = \frac{(s-2)e^{-s}}{s^2 - 4s + 3}$$

Solution

Factor the denominator.

$$F(s) = \frac{s-2}{(s-3)(s-1)} e^{-s}$$

Partially decompose the fraction.

$$F(s) = \left(\frac{\frac{1}{2}}{s-3} + \frac{\frac{1}{2}}{s-1} \right) e^{-s}$$

Take the inverse Laplace transform now to get $f(t)$.

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}\{F(s)\} \\ &= \mathcal{L}^{-1}\left\{ \left(\frac{\frac{1}{2}}{s-3} + \frac{\frac{1}{2}}{s-1} \right) e^{-s} \right\} \\ &= \left[\frac{1}{2}e^{3(t-1)} + \frac{1}{2}e^{(t-1)} \right] H(t-1) \\ &= \frac{1}{2} \left[e^{3(t-1)} + e^{t-1} \right] H(t-1) \\ &= \frac{1}{2}e^{2(t-1)} \left[e^{(t-1)} + e^{-(t-1)} \right] H(t-1) \\ &= e^{2(t-1)} \cosh(t-1)u_1(t) \end{aligned}$$