

Problem 25

Suppose that $F(s) = \mathcal{L}\{f(t)\}$ exists for $s > a \geq 0$.

(a) Show that if c is a positive constant, then

$$\mathcal{L}\{f(ct)\} = \frac{1}{c}F\left(\frac{s}{c}\right), \quad s > ca.$$

(b) Show that if k is a positive constant, then

$$\mathcal{L}^{-1}\{F(ks)\} = \frac{1}{k}f\left(\frac{t}{k}\right).$$

(c) Show that if a and b are constants with $a > 0$, then

$$\mathcal{L}^{-1}\{F(as + b)\} = \frac{1}{a}e^{-bt/a}f\left(\frac{t}{a}\right).$$