Problem 26

In each of Problems 26 through 29, use the results of Problem 25 to find the inverse Laplace transform of the given function.

$$F(s) = \frac{2^{n+1}n!}{s^{n+1}}$$

Solution

Rewrite the right side.

$$\begin{aligned} F(s) &= \left(\frac{2}{s}\right)^{n+1} n! \\ &= \frac{n!}{\left(\frac{s}{2}\right)^{n+1}} \end{aligned}$$

Apply the two transforms,

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \quad \text{and} \quad c\mathcal{L}\{f(ct)\} = F\left(\frac{s}{c}\right),$$

together to get $f(t)$.

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}\left\{\frac{n!}{\left(\frac{s}{2}\right)^{n+1}}\right\} \\ &= 2(2t)^n \end{aligned}$$