# Problem 38

In each of Problems 35 through 38, use the result of Problem 34 to find the Laplace transform of the given function.

$$f(t) = \sin t, \qquad 0 \le t < \pi;$$
$$f(t+\pi) = f(t).$$
See Figure 6.3.10.

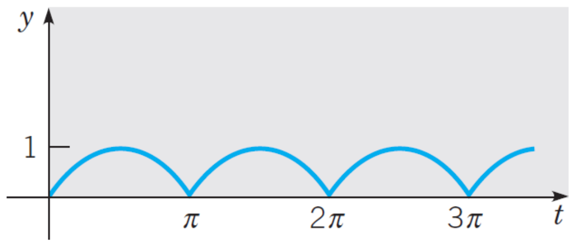


**FIGURE 6.3.10** The function $f(t)$ in Problem 38; a rectified sine wave.

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### Solution

For a function that repeats itself periodically every $T$ units, the Laplace transform is

$$\mathcal{L}\{f(t)\} = \frac{\int_0^T e^{-st} f(t)\,dt}{1 - e^{-sT}}.$$

In this problem the period is $T = \pi$.

$$\mathcal{L}\{f(t)\} = \frac{\int_0^\pi e^{-st}(\sin t)\,dt}{1 - e^{-\pi s}} \tag{1}$$

Use integration by parts twice to evaluate the integral in the numerator.

$$\begin{aligned}
\int_0^\pi e^{-st}(\sin t)\,dt &= \int_0^\pi e^{-st}\frac{d}{dt}(-\cos t)\,dt \\
&= e^{-st}(-\cos t)\Big|_0^\pi - \int_0^\pi (-s)e^{-st}(-\cos t)\,dt \\
&= e^{-\pi s} + 1 - s\int_0^\pi e^{-st}\cos t\,dt \\
&= e^{-\pi s} + 1 - s\int_0^\pi e^{-st}\frac{d}{dt}(\sin t)\,dt \\
&= e^{-\pi s} + 1 - s\left[e^{-st}(\sin t)\Big|_0^\pi - \int_0^\pi (-s)e^{-st}(\sin t)\,dt\right] \\
&= e^{-\pi s} + 1 - s^2\int_0^\pi e^{-st}(\sin t)\,dt
\end{aligned}$$

As a result,

$$(1+s^2)\int_0^\pi e^{-st}(\sin t)\,dt = e^{-\pi s}+1$$

$$\int_0^\pi e^{-st}(\sin t)\,dt = \frac{e^{-\pi s}+1}{s^2+1},$$

and equation (1) becomes

$$\begin{aligned}\mathcal{L}\{f(t)\} &= \frac{\frac{e^{-\pi s}+1}{s^2+1}}{1-e^{-\pi s}} \\ &= \frac{e^{-\pi s}+1}{(s^2+1)(1-e^{-\pi s})}.\end{aligned}$$