Problem 15

Find an expression involving $u_c(t)$ for a function g that ramps up from zero at $t = t_0$ to the value h at $t = t_0 + k$ and then ramps back down to zero at $t = t_0 + 2k$.

Solution

$$g(t) = \frac{h}{k}(t - t_0)H(t - t_0) - \frac{h}{k}(t - t_0)H(t - t_0 - k) + \frac{h}{k}(t_0 + 2k - t)H(t - t_0 - k) - \frac{h}{k}(t_0 + 2k - t)H(t - t_0 - 2k)$$

For the first term, the Heaviside function $H(t - t_0)$, defined to be 1 if $t > t_0$ and 0 if $t < t_0$, makes the function start increasing at $t = t_0$. The factor of $(h/k)(t - t_0)$ makes it so that it reaches a height of h at $t = t_0 + k$. The second term, which activates when $t = t_0 + k$, cancels the first term. The third term makes the function start decreasing at $t = t_0$. The factor of $(h/k)(t_0 + 2k - t)$ makes it so that $f(t)$ falls to zero at $t = t_0 + 2k$. The fourth term, which activates at $t = t_0 + 2k$, cancels the third term. $g(t)$ can also be written as

$$\begin{aligned} g(t) &= \frac{h}{k}(t - t_0)[\underbrace{H(t - t_0)}_{\text{turn on}} - \underbrace{H(t - t_0 - k)}_{\text{turn off}}] + \frac{h}{k}(t_0 + 2k - t)[\underbrace{H(t - t_0 - k)}_{\text{turn on}} - \underbrace{H(t - t_0 - 2k)}_{\text{turn off}}] \\ &= \frac{h}{k}(t - t_0)[u_{t_0}(t) - u_{t_0+k}(t)] + \frac{h}{k}(t_0 + 2k - t)[u_{t_0+k}(t) - u_{t_0+2k}(t)]. \end{aligned}$$

Below is a sample of $g(t)$ versus t if $h = 1$, $k = 2$, and $t_0 = 3$.