

Problem 16

A certain spring-mass system satisfies the initial value problem

$$u'' + \frac{1}{4}u' + u = kg(t), \quad u(0) = 0, \quad u'(0) = 0,$$

where $g(t) = u_{3/2}(t) - u_{5/2}(t)$ and $k > 0$ is a parameter.

- (a) Sketch the graph of $g(t)$. Observe that it is a pulse of unit magnitude extending over one time unit.
- (b) Solve the initial value problem.
- (c) Plot the solution for $k = 1/2$, $k = 1$, and $k = 2$. Describe the principal features of the solution and how they depend on k .
- (d) Find, to two decimal places, the smallest value of k for which the solution $u(t)$ reaches the value 2.
- (e) Suppose $k = 2$. Find the time τ after which $|u(t)| < 0.1$ for all $t > \tau$.