Problem 17

Modify the problem in Example 2 of this section by replacing the given forcing function $g(t)$ by

$$f(t) = \left[u_5(t)(t-5) - u_{5+k}(t)(t-5-k)\right]/k.$$

(a) Sketch the graph of $f(t)$ and describe how it depends on k. For what value of k is $f(t)$ identical to $g(t)$ in the example?

(b) Solve the initial value problem

$$y'' + 4y = f(t), \qquad y(0) = 0, \quad y'(0) = 0.$$

(c) The solution in part (b) depends on k, but for sufficiently large t the solution is always a simple harmonic oscillation about $y = 1/4$. Try to decide how the amplitude of this eventual oscillation depends on k. Then confirm your conclusion by plotting the solution for a few different values of k.