

## Problem 18

Consider the initial value problem

$$y'' + \frac{1}{3}y' + 4y = f_k(t), \quad y(0) = 0, \quad y'(0) = 0,$$

where

$$f_k(t) = \begin{cases} 1/2k, & 4 - k \leq t < 4 + k \\ 0, & 0 \leq t < 4 - k \quad \text{and} \quad t \geq 4 + k \end{cases}$$

and  $0 < k < 4$ .

- (a) Sketch the graph of  $f_k(t)$ . Observe that the area under the graph is independent of  $k$ . If  $f_k(t)$  represents a force, this means that the product of the magnitude of the force and the time interval during which it acts does not depend on  $k$ .
- (b) Write  $f_k(t)$  in terms of the unit step function and then solve the given initial value problem.
- (c) Plot the solution for  $k = 2$ ,  $k = 1$ , and  $k = \frac{1}{2}$ . Describe how the solution depends on  $k$ .