

## Problem 19

Consider the initial value problem

$$y'' + y = f(t), \quad y(0) = 0, \quad y'(0) = 0,$$

where

$$f(t) = u_0(t) + 2 \sum_{k=1}^n (-1)^k u_{k\pi}(t).$$

- (a) Draw the graph of  $f(t)$  on an interval such as  $0 \leq t \leq 6\pi$ .
- (b) Find the solution of the initial value problem.
- (c) Let  $n = 15$  and plot the graph of the solution for  $0 \leq t \leq 60$ . Describe the solution and explain why it behaves as it does.
- (d) Investigate how the solution changes as  $n$  increases. What happens as  $n \rightarrow \infty$ ?