

Problem 25

- (a) By the method of variation of parameters, show that the solution of the initial value problem

$$y'' + 2y' + 2y = f(t); \quad y(0) = 0, \quad y'(0) = 0$$

is

$$y = \int_0^t e^{-(t-\tau)} f(\tau) \sin(t - \tau) d\tau.$$

- (b) Show that if $f(t) = \delta(t - \pi)$, then the solution of part (a) reduces to

$$y = u_\pi(t) e^{-(t-\pi)} \sin(t - \pi).$$

- (c) Use a Laplace transform to solve the given initial value problem with $f(t) = \delta(t - \pi)$, and confirm that the solution agrees with the result of part (b).