Problem 2

Find an example different from the one in the text showing that $(f * 1)(t)$ need not be equal to $f(t)$.

Solution

The convolution of two functions, f and g, is defined to be

$$f * g = \int_0^t f(t-\tau)g(\tau)\,d\tau.$$

If $g = 1$, then

$$f * 1 = \int_0^t f(t-\tau)\,d\tau.$$

In the text, the case $f(t) = \cos t$ was considered to show that $f * 1 \neq f$.

$$\begin{aligned} f * 1 &= \int_0^t \cos(t-\tau)\,d\tau \\ &= -\sin(t-\tau)\Big|_0^t \\ &= \sin t - \sin 0 \\ &= \sin t \\ &\neq f \end{aligned}$$

Check out the case $f(t) = \sin t$.

$$\begin{aligned} f * 1 &= \int_0^t \sin(t-\tau)\,d\tau \\ &= \cos(t-\tau)\Big|_0^t \\ &= \cos 0 - \cos t \\ &= 1 - \cos t \\ &\neq f \end{aligned}$$

Also, check out the case $f(t) = e^t$.

$$\begin{aligned} f * 1 &= \int_0^t e^{t-\tau}\,d\tau \\ &= -e^{t-\tau}\Big|_0^t \\ &= e^t - e^0 \\ &= e^t - 1 \\ &\neq f \end{aligned}$$