

Problem 5

In each of Problems 4 through 7, find the Laplace transform of the given function.

$$f(t) = \int_0^t e^{-(t-\tau)} \sin \tau \, d\tau$$

Solution

Recognize that $f(t)$ is a convolution integral of the two functions, e^{-t} and $\sin t$. The Laplace transform of $f(t)$ can be found by using the convolution theorem, which states that

$$\mathcal{L} \left\{ \int_0^t g(t-\tau)h(\tau) \, d\tau \right\} = G(s)H(s),$$

where G and H are the Laplace transforms of g and h , respectively. Therefore,

$$\begin{aligned} \mathcal{L}\{f(t)\} &= \mathcal{L} \left\{ \int_0^t e^{-(t-\tau)} \sin \tau \, d\tau \right\} \\ &= \mathcal{L}\{e^{-t}\} \mathcal{L}\{\sin t\} \\ &= \left(\frac{1}{s+1} \right) \left(\frac{1}{s^2+1} \right) \\ &= \frac{1}{(s+1)(s^2+1)}. \end{aligned}$$