

Problem 12

- (a) If $f(t) = t^m$ and $g(t) = t^n$, where m and n are positive integers, show that

$$f * g = t^{m+n+1} \int_0^1 u^m (1-u)^n du.$$

- (b) Use the convolution theorem to show that

$$\int_0^1 u^m (1-u)^n du = \frac{m!n!}{(m+n+1)!}.$$

- (c) Extend the result of part (b) to the case where m and n are positive numbers but not necessarily integers.