

Problem 22

Consider the Volterra integral equation (see Problem 21)

$$\phi(t) + \int_0^t (t - \xi)\phi(\xi) d\xi = \sin 2t. \quad (\text{i})$$

- (a) Solve the integral equation (i) by using the Laplace transform.
- (b) By differentiating Eq. (i) twice, show that $\phi(t)$ satisfies the differential equation

$$\phi''(t) + \phi(t) = -4 \sin 2t.$$

Show also that the initial conditions are

$$\phi(0) = 0, \quad \phi'(0) = 2.$$

- (c) Solve the initial value problem in part (b), and verify that the solution is the same as the one in part (a).