Problem 8

In each of Problems 8 through 11, find the inverse Laplace transform of the given function by using the convolution theorem.

$$F(s) = \frac{1}{s^4(s^2+1)}$$

Solution

Recognize that $F(s)$ is a product of the two Laplace transforms,

$$\frac{1}{s^4} = \frac{1}{6}\frac{3!}{s^{3+1}} = \mathcal{L}\left\{\frac{1}{6}t^3\right\} \quad \text{and} \quad \frac{1}{s^2+1} = \mathcal{L}\{\sin t\}.$$

According to the convolution theorem, the inverse Laplace transform of a product is a convolution integral.

$$\mathcal{L}^{-1}\left\{G(s)H(s)\right\} = \int_0^t g(t-\tau)h(\tau)\,d\tau$$

g and h are the inverse Laplace transforms of G and H, respectively. Therefore,

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}\{F(s)\} \\ &= \mathcal{L}^{-1}\left\{\frac{1}{s^4(s^2+1)}\right\} \\ &= \int_0^t \frac{1}{6}(t-\tau)^3 \sin\tau\,d\tau. \end{aligned}$$