

Exercise 4

According to Sec. 9, the three cube roots of a nonzero complex number z_0 can be written c_0 , $c_0\omega_3$, $c_0\omega_3^2$ where c_0 is the principal cube root of z_0 and

$$\omega_3 = \exp\left(i\frac{2\pi}{3}\right) = \frac{-1 + \sqrt{3}i}{2}.$$

Show that if $z_0 = -4\sqrt{2} + 4\sqrt{2}i$, then $c_0 = \sqrt{2}(1 + i)$ and the other two cube roots are, in rectangular form, the numbers

$$c_0\omega_3 = \frac{-(\sqrt{3} + 1) + (\sqrt{3} - 1)i}{\sqrt{2}}, \quad c_0\omega_3^2 = \frac{(\sqrt{3} - 1) - (\sqrt{3} + 1)i}{\sqrt{2}}.$$