

Exercise 15

Follow the steps below to give an algebraic derivation of the triangle inequality (Sec. 4)

(a) Show that

$$|z_1 + z_2|^2 = (z_1 + z_2)(\bar{z}_1 + \bar{z}_2) = z_1\bar{z}_1 + (z_1\bar{z}_2 + \overline{z_1\bar{z}_2}) + z_2\bar{z}_2.$$

(b) Point out why

$$z_1\bar{z}_2 + \overline{z_1\bar{z}_2} = 2\operatorname{Re}(z_1\bar{z}_2) \leq 2|z_1||z_2|.$$

(c) Use the results in parts (a) and (b) to obtain the inequality

$$|z_1 + z_2|^2 \leq (|z_1| + |z_2|)^2,$$

and note how the triangle inequality follows.