

Exercise 3

Verify properties (3) and (4) of conjugates in Sec. 5.

Solution

Suppose that $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$.

Verification of Property (3)

$$\begin{aligned}\overline{z_1 - z_2} &= \overline{(x_1 + iy_1) - (x_2 + iy_2)} \\ &= \overline{(x_1 - x_2) + i(y_1 - y_2)} \\ &= (x_1 - x_2) - i(y_1 - y_2) \\ &= x_1 - iy_1 - (x_2 - iy_2) \\ &= z_1 - z_2\end{aligned}$$

Verification of Property (4)

$$\begin{aligned}\overline{z_1 z_2} &= \overline{(x_1 + iy_1)(x_2 + iy_2)} \\ &= \overline{x_1 x_2 - y_1 y_2 + i(x_1 y_2 + x_2 y_1)} \\ &= x_1 x_2 - y_1 y_2 - i(x_1 y_2 + x_2 y_1) \\ &= (x_1 - iy_1)(x_2 - iy_2) \\ &= \overline{x_1 + iy_1} \overline{x_2 + iy_2} \\ &= z_1 z_2\end{aligned}$$