Exercise 11

(*a*) Use the binomial formula (Sec. 3) and de Moivre's formula (Sec. 7) to write

$$\cos n\theta + i\sin n\theta = \sum_{k=0}^{n} \binom{n}{k} \cos^{n-k}\theta (i\sin\theta)^k \qquad (n = 0, 1, 2, \ldots).$$

Then define the integer m by means of the equations

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n-1)/2 & \text{if } n\text{is odd} \end{cases}$$

and use the above summation to show that [compare with Exercise 10(*a*)]

$$\cos n\theta = \sum_{k=0}^{m} \binom{n}{2k} (-1)^k \cos^{n-2k}\theta \sin^{2k}\theta \qquad (n = 0, 1, 2, \ldots).$$

(*b*) Write $x = \cos\theta$ in the final summation in part (*a*) to show that it becomes a polynomial

$$T_n(x) = \sum_{k=0}^{m} \binom{n}{2k} (-1)^k x^{n-2k} (1 - x^2)^k$$

of degree n ($n = 0, 1, 2, \ldots$) in the variable x.*

[TYPO: A space is needed between "n" and "is." Oddly, this mistake wasn't in the 7th edition.]

*These are called Chebyshev polynomials and are prominent in approximation theory.