

## Exercise 9

Establish the identity

$$1 + z + z^2 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z} \quad (z \neq 1)$$

and then use it to derive *Lagrange's trigonometric identity*:

$$1 + \cos \theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1}{2} + \frac{\sin [(2n + 1)\theta/2]}{2 \sin(\theta/2)} \quad (0 < \theta < 2\pi).$$

*Suggestion:* As for the first identity, write  $S = 1 + z + z^2 + \cdots + z^n$  and consider the difference  $S - zS$ . To derive the second identity, write  $z = e^{i\theta}$  in the first one.