

Exercise 3

In Exercises 1 through 4, take the indented contour in Fig. 101 (Sec. 82).

Use the function

$$f(z) = \frac{z^{1/3} \log z}{z^2 + 1} = \frac{e^{(1/3) \log z} \log z}{z^2 + 1} \quad \left(|z| > 0, -\frac{\pi}{2} < \arg z < \frac{3\pi}{2} \right)$$

to derive this pair of integration formulas:

$$\int_0^{\infty} \frac{\sqrt[3]{x} \ln x}{x^2 + 1} dx = \frac{\pi^2}{6}, \quad \int_0^{\infty} \frac{\sqrt[3]{x}}{x^2 + 1} dx = \frac{\pi}{\sqrt{3}}.$$