Exercise 5

Use the function

$$f(z) = \frac{z^{1/3}}{(z+a)(z+b)} = \frac{e^{(1/3)\log z}}{(z+a)(z+b)} \qquad (|z| > 0,\ 0 < \arg z < 2\pi)$$

and a closed contour similar to the one in Fig. 103 (Sec. 84) to show formally that

$$\int_0^\infty \frac{\sqrt[3]{x}}{(x+a)(x+b)}\,dx = \frac{2\pi}{\sqrt{3}} \cdot \frac{\sqrt[3]{a} - \sqrt[3]{b}}{a-b} \qquad (a > b > 0).$$