Exercise 15

The transverse vibration of an infinite elastic beam of mass m per unit length and bending stiffness EI is governed by

$$u_{tt} + a^2 u_{xxxx} = 0, \quad a^2 = \frac{EI}{m}, \ -\infty < x < \infty, \ t > 0.$$

Solve this equation subject to the boundary and initial data

$$\begin{aligned} &u(0,t) = 0 \quad \text{for all } t > 0, \\ &u(x,0) = \phi(x), \quad \text{and} \quad u_t(x,0) = \psi''(x) \quad \text{for } 0 < x < \infty. \end{aligned}$$

Show that the Fourier transform solution is

$$U(k,t) = \Phi(k)\cos(atk^2) - \left(\frac{1}{a}\right)\Psi(k)\sin(atk^2).$$

Find the integral solution for $u(x,t)$.