

Exercise 16

Solve the Lamb (1904) problem in geophysics that satisfies the Helmholtz equation in an infinite elastic half-space

$$u_{xx} + u_{zz} + \frac{\omega^2}{c_2^2}u = 0, \quad -\infty < x < \infty, \quad z > 0,$$

where ω is the frequency and c_2 is the shear wave speed.

At the surface of the half-space ($z = 0$), the boundary condition relating the surface stress to the impulsive point load distribution is given by

$$\mu \frac{\partial u}{\partial z} = -P\delta(x) \quad \text{at } z = 0,$$

where μ is one of the Lamé constants, P is a constant, and

$$u(x, z) \rightarrow 0 \quad \text{as } z \rightarrow \infty \text{ for } -\infty < x < \infty.$$

Show that the solution in terms of polar coordinates is

$$\begin{aligned} u(x, z) &= \frac{P}{2i\mu} H_0^{(2)}\left(\frac{\omega r}{c_2}\right) \\ &\sim \frac{P}{2i\mu} \left(\frac{2c_2}{\pi\omega r}\right)^{\frac{1}{2}} \exp\left(\frac{\pi i}{4} - \frac{i\omega r}{c_2}\right) \quad \text{for } \omega r \gg c_2. \end{aligned}$$