

Exercise 17

Find the solution of the Cauchy-Poisson problem (Debnath 1994, p. 83) in inviscid water of infinite depth which is governed by

$$\begin{aligned}\phi_{xx} + \phi_{zz} &= 0, & -\infty < x < \infty, & -\infty < z \leq 0, & t > 0, \\ \left. \begin{aligned}\phi_z - \eta_t &= 0, \\ \phi_t + g\eta &= 0\end{aligned}\right\} & \text{on } z = 0, & t > 0, \\ \phi_z &\rightarrow 0 & \text{as } z \rightarrow -\infty. \\ \phi(x, 0, 0) &= 0, & \text{and } \eta(x, 0) &= P\delta(x),\end{aligned}$$

where $\phi = \phi(x, z, t)$ is the velocity potential, $\eta(x, t)$ is the free surface elevation, and P is a constant.

Derive the asymptotic solution for the free surface elevation as $t \rightarrow \infty$.