Exercise 18

Obtain the solutions for the velocity potential $\phi(x,z,t)$ and the free surface elevation $\eta(x,t)$ involved in the two-dimensional surface waves in water of finite (or infinite) depth h. The governing equation, boundary, and free surface conditions and initial conditions (see Debnath 1994, p. 92) are

$$\phi_{xx} + \phi_{zz} = 0, \quad -h \le z \le 0, \ -\infty < x < \infty, \ t > 0,$$

$$\left.\begin{aligned} &\phi_t + g\eta = -\frac{P}{\rho} p(x) \exp(i\omega t), \\ &\phi_z - \eta_t = 0 \end{aligned}\right\} \quad z = 0, \ t > 0,$$

$$\phi(x,z,0) = 0 = \eta(x,0) \quad \text{for all } x \text{ and } z.$$