

## Exercise 19

Solve the steady-state surface wave problem (Debnath 1994, p. 47) on a running stream of infinite depth due to an external steady pressure applied to the free surface. The governing equation and the free surface conditions are

$$\begin{aligned} \phi_{xx} + \phi_{zz} &= 0, & -\infty < x < \infty, & -\infty < z < 0, & t > 0, \\ \left. \begin{aligned} \phi_x + U\phi_x + g\eta &= -\frac{P}{\rho}\delta(x)\exp(\varepsilon t), \\ \eta_t + U\eta_x &= \phi_z \end{aligned} \right\} & \text{on } z = 0 \text{ } (\varepsilon > 0), \\ \phi_z &\rightarrow 0 & \text{as } z &\rightarrow -\infty. \end{aligned}$$

where  $U$  is the stream velocity,  $\phi(x, z, t)$  is the velocity potential, and  $\eta(x, t)$  is the free surface elevation. [TYPO: This should be  $\phi_t$ !]