

**Exercise 21**

Use the Fourier transform to solve the boundary-value problem

$$u_{xx} + u_{yy} = -x \exp(-x^2), \quad -\infty < x < \infty, \quad 0 < y < \infty,$$

$u(x, 0) = 0$ , for  $-\infty < x < \infty$ ,  $u$  and its derivative vanish as  $y \rightarrow \infty$ . Show that

$$u(x, y) = \frac{1}{\sqrt{4\pi}} \int_0^\infty [1 - \exp(-ky)] \frac{\sin(kx)}{k} \exp\left(-\frac{k^2}{4}\right) dk.$$